# Einstein Workshop on Discrete Geometry and Topology

Titles, abstracts, and posters

March 13 to March 16 2018, Freie Universiät Berlin

## Contents

# 1 Tuesday

## 1.1 Bernd Sturmfels

Date: Tuesday 13th, 09:00–09:45
Title: *The Geometry of Gaussoids*

Gaussoids offer a new link between combinatorics, statistics and algebraic geometry. Introduced by Lnenicka and Matus in 2007, their axioms describe conditional independence for Gaussian random variables. We explain this theory and how it relates to matroids. The role of the Grassmannian for matroids is now played by a projection of the Lagrangian Grassmannian. We discuss the classification and realizability of gaussoids, and we explore oriented gaussoids, valuated gaussoids, and the analogue to positroids.

This is joint work with Tobias Boege, Alessio D'Ali and Thomas Kahle.

## 1.2 Gaku Liu

Date: Tuesday 13th, 10:15–11:00
Title: *Near-rational subdivisions of non-rational polytopes*

It is an easy fact that any rational polytope can be subdivided into simplices which are rational. In this talk we prove that any polytope with rational slopes can be subdivided into products of simplices with rational slopes, with vertex coordinates in the same $Q$-span as the vertex coordinates of the polytope. This extends results in toric geometry from affine monoids to valuative monoids, and has implications in log geometry. (No algebraic geometry will be used in the talk.)

This is joint work in progress with Karim Adiprasito, Igor Pak, and Michael Temkin.

## 1.3 Katharina Jochemko

Date: Tuesday 13th, 11:15–12:00
Title: *$h^*$-polynomials of dilated lattice polytopes*

Interlacing polynomials are a powerful method to prove that a polynomial has only real roots. In this talk I present applications to $h^*$-polynomials of dilated lattice polytopes.

## 1.4 Tristram Bogart

Date: Tuesday 13th, 16:00–16:45
Title: *Parametric Presburger Arithmetic*

Parametric Presburger arithmetic concerns families of sets defined using quantifiers and Boolean combinations of formulas of the form $a(t) \cdot x \leq b(t)$, where $a(t) \in Z[t]$, $d, b(t) \in Z[t]$. In the quantifier-free form these formulas represent sets of integer points in families of (unions of) polyhedra with polynomially varying facet inequalities. With quantifiers, they may represent more general sets such as polynomially varying families of numerical semigroups.

Recent results of Chen, Li, and Sam; Calegari and Walker; Roune and Woods; and Shen concern specific families in parametric Presburger arithmetic that exhibit quasi-polynomial behavior. For example, $|S_t|$ might be a quasi-polynomial function of $t$ or an element $x(t) \in S_t$ might be specifiable as a function with quasi-polynomial coordinates for sufficiently large $t$. Woods conjectured that all parametric Presburger sets exhibit this quasi-polynomial behavior. With Goodrick and Woods, we proved this conjecture, using various tools from logic and combinatorics.

Analogously, $k$-parametric Presburger arithmetic concerns families of sets defined in the same way but with $Z[t]$ replaced by $Z[t_1, \ldots, t_k]$. Building on recent work of Nguyen and Pak, we prove with Goodrick, Nguyen and Woods the following result in stark contrast to the 1-parameter case: if P $\neq$ NP, then there are 2-parametric Presburger families whose counting function $|S(t_1, t_2)|$ cannot even be evaluated in polynomial time.

## 1.5 Problem Session

Date: Tuesday 13th, 17:00–17:45

# 2 Wednesday

## 2.1 Monika Ludwig

Date: Wednesday 14th, 9:00–09:45
Title: *Minkowski Valuations on Polytopes*

A function $Z$ from a family $\mathcal{F}$ of subsets of $\mathbb{R}^n$ with values in an abelian group is a valuation if

$$Z(P) + Z(Q) = Z(P \cup Q) + Z(P \cap Q)$$

whenever $P, Q, P \cup Q, P \cap Q \in \mathcal{F}$ and $Z(\emptyset) = 0$. If the codomain is $\mathcal{K}^n$, the set of convex bodies in $\mathbb{R}^n$, and addition is Minkowski addition, then we call $Z$ a Minkowski valuation. The operator that asscociates with a polytope its projection body and its difference body are important Minkowski valuations.

In the talk, new and old Minkowski valuations are presented along with classification results and applications in geometric inequalities.

## 2.2 Bernardo González-Merino

Date: Wednesday 14th, 10:15–11:00
Title: *Optimizing volume with prescribed diameter or minimum width*

Some classical questions of Convex Geometry, such as optimizations of the volume (or Lebesgue measure) with prescribed diameter or minimum width, are still open. The aim of this talk is to survey on this topic, remembering the main achievements and collecting some recent results in this area.

## 2.3 Eugenia Saorín Gómez

Date: Wednesday 14th, 11:15–12:00
Title: *Some modern aspects of classical Convex Geometry*

The aim of this talk is to report on some modern aspects of classical Convex Geometry, in particular, of the Brunn-Minkowski Theory.

The theory of Convex Bodies (convex and compact sets of $\mathbb{R}^n$) in which the Brunn-Minkowski Theory is originally framed on, is a classical central theme in metric geometry. Convex Geometry, as the geometry of convex domains in the Euclidean space, has inherent geometric and analytic connections, as well as further links with other fields within Mathematics, and beyond.

The Brunn-Minkowski theory is based on the combination of the Minkowski sum of convex sets with the notion of volume (Lebesgue measure). Classical notions in this context are mixed volumes, in particular, quermassintegrals; geometric inequalities, especially the Brunn-Minkowski inequality, as well as the equality cases of them; and polynomial expansions, as the Steiner formula.

In the last two decades, the classical theory of convex bodies has been fruitfully enriched, expanded and generalized. The replacement of the Minkowski sum by other combinations of convex bodies (or more general sets), the interplay of convex sets and functions or the theory of valuations on the space of convex bodies are some examples of these successful and far-reaching new aspects of Convex Geometry.

In the classical theory there is, though, room for further investigation, as there are still some open questions. Many of them are concerned with equality cases of inequalities on convex bodies, and improvements of these inequalities. Some other ask for geometric aspects of the structure of convex bodies. In this talk we will concentrate on three aspects of the classical theory of convex bodies for which there are still some questions to answer. For each of them, a short introduction providing the necessary background and the state of art of the questions will be provided. In particular we will deal with some aspects of the Steiner polynomial and the Brunn-Minkowski inequality. Then, we will focus on decompositions (and *approximations*) of convex bodies by means of Minkowski sums of related convex bodies. Finally we will discuss some aspects of the more modern $L_p$-theory of convex bodies.

## 2.4 Martina Juhnke-Kubitzke

Date: Wednesday 14th, 16:00–16:45
Title: *Balanced shellings on manifolds*

A classical result by Pachner states that any two PL homoeomorphic manifolds with boundary are related by a sequence of shellings and inverse shellings. We show that, for balanced manifolds, such a sequence can be chosen in such a way that in each step balancedness is preserved.

This is joint work with Lorenzo Venturello.

## 2.5 Karim Adiprasito

Date: Wednesday 14th, 17:00–17:45
Title: *Combinatorics and geometry of extremal and close-to-extremal $g$-vectors*

# 3 Thursday

## 3.1 Eric Sedgwick

Date: Thursday 15th, 09:00–09:45
Title: *Hard problems in* 3*-manifold topology*

We show that the following problems are NP-hard: a) the TRIVIAL SUBLINK PROBLEM, the problem of deciding whether a link in the 3-sphere has an $n$ component trivial sub-link, and b) the UNLINKING PROBLEM, the problem of deciding whether a link in the 3-sphere can be made trivial by changing $n$ crossings. This work is related to the authors' recent result that EMBED2$\rightarrow$3, the problem of deciding whether a 2-complex embeds in 3-space, is NP-hard.

This is joint work with Arnaud de Mesmay, Yo'av Rieck and Martin Tancer.

## 3.2 Uli Wagner

Date: Thursday 15th, 10:15–11:00
Title: *Shellability is NP-complete*

Consider a pure 2-dimensional simplicial complex $X$ (in other words, a 3-uniform hypergraph). A shelling of $X$ is an inductive procedure of building $X$ by adding the triangles of $X$ one at a time, in such a way that each new triangle (except the first one) intersects the previously constructed complex in a connected 1-dimensional piece (consisting of one, two, or all three edges on the boundary of the triangle); $X$ is called shellable if has a shelling. The definition naturally generalizes to higher-dimensional complexes.

Shellings and shellability are fundamental concepts in a variety of areas including polytope theory (the boundary of every convex polytope is shellable, by a theorem of Brugesser and Mani), piecewise-linear topology, topological combinatorics, and others. One reason for its importance is that shellability – a purely combinatorial property – has strong topological consequences. For instance if a $d$-dimensional complex $X$ is shellable and moreover a pseudomanifold (i.e., if every $(d-1)$-simplex of $X$ contained in exactly two $d$-simplices, which is easily checked) then $X$ is homeomorphic to a $d$-dimensional sphere – a property that is algorithmically undecidable for $d \geq 5$.

From a computational viewpoint, it is a natural question (raised at least as early as in the 1970's by Danaraj and Klee) whether one can decide efficiently whether a given complex is shellable. We settle this in the negative (modulo P≠NP) and show that shellability of $d$-dimensional complexes is NP-complete for every $d \geq 2$.

Joint work with Xavier Goaoc, Pavel Paták, Zuzana Patáková, and Martin Tancer.

## 3.3 Ulrich Bauer

Date: Thursday 15th, 11:15–12:00
Title: *The Reeb Graph Edit Distance is Universal*

We consider the setting of Reeb graphs of piecewise linear functions and study distances between them that are stable, meaning that functions which are similar in the supremum norm ought to have similar Reeb graphs. We define an edit distance for Reeb graphs and prove that it is stable and universal, meaning that it provides an upper bound to any other stable distance. In contrast, via a specific construction, we show that the interleaving distance and the functional distortion distance on Reeb graphs are not universal.

This is joint work with Claudia Landi and Facundo Mémoli.

# 4 Friday

## 4.1 Isabella Novik

Date: Friday 16th, 09:00–09:45
Title: *Mysteries around the edge numbers of centrally symmetric polytopes*

We will discuss several recent results as well as the remaining mysteries surrounding the upper and lower bounds on the number of edges that a centrally symmetric simplicial polytope with $N$ vertices can have.

### 4.2 Guillermo Pineda-Villavicencio

Date: Friday 16th, 10:15–11:00
Title: *Three questions on graphs of polytopes*

The first part of the talk is about reconstructing the face lattice of a polytope from low-dimensional skeletons. The $k$-skeleton of a polytope is the set of all its faces of dimension at most $k$.

Blind and Mani, and later Kalai, showed that the face lattice of a simple polytope is determined by its graph, namely its 1-skeleton.

Call a vertex of a $d$-polytope non-simple if the number of edges incident to it is greater than $d$.

We show the following.

1. The face lattice of any $d$-polytope with at most two non-simple vertices is determined by its 1-skeleton;
2. the face lattice of any $d$-polytope with at most $(d-2)$ non-simple vertices is determined by its 2-skeleton; and
3. for any $d > 3$ there are two $d$-polytopes with $(d-1)$ nonsimple vertices, isomorphic $(d-3)$-skeleton and nonisomorphic face lattices.

The second part of the talk explores two questions on the connectivity of graphs of cubical polytopes. A cubical $d$-polytope is a polytope with all its facets being combinatorially equivalent to $(d-1)$-cubes.

Firstly, we establish the following.

4. For any integer $d \geq 3$ and any nonnegative integer $\alpha \leq d-3$, the graph of a cubical $d$-polytope with minimum degree at least $d+\alpha$ is $(d+\alpha)$-connected; and
5. for any integer $d \geq 4$ and any nonnegative integer $\alpha \leq d-3$, every separator of cardinality $d+\alpha$ in such a graph consists of all the neighbours of some vertex and breaks the polytope into exactly two components.

The second question is about the linkedness of cubical polytopes. A graph with at least $2k$ vertices is $k$-linked if, for every set of $2k$ distinct vertices organised in arbitrary $k$ pairs of vertices, there are $k$ vertex-disjoint paths joining the vertices in the pairs. In this direction, the following is our main contribution.

6. Cubical $d$-polytopes are $\lfloor (d+1)/2 \rfloor$-linked for every $d \neq 3$; this is the maximum possible linkedness for such a class of polytopes.

## 4.3 Eran Nevo

Date: Friday 16th, 11:15–12:00
Title: *Face numbers of cubical polytopes*

We will discuss face numbers of cubical polytopes. In particular, regarding the question what is the minimal closed cone containing all $f$-vectors of cubical $d$-polytopes, we construct cubical polytopes showing that this cone contains Adin's nonnegative $g$-orthant, thus verifying one direction of the Cubical Generalized Lower Bound Conjecture of Babson, Billera and Chan.

Based on joint work with Ron Adin and Daniel Kalmanovich.

## 4.4 Alexander Postnikov

Date: Friday 16th, 16:00–16:45
Title: *Positive Grassmannian and polyhedral subdivisions*

The nonnegative Grassmannian is a cell complex with rich geometric, algebraic, and combinatorial structures. Its study involves interesting combinatorial objects, such as positroids and plabic graphs. Remarkably, the same combinatorial structures appeared in many other areas of mathematics and physics, e.g., in the study of cluster algebras, scattering amplitudes, and solitons. We discuss new ways to think about these structures. In particular, we identify plabic graphs and more general Grassmannian graphs with polyhedral subdivisions induced by 2-dimensional projections of hypersimplices. This implies a close relationship between the positive Grassmannian and the theory of fiber polytopes and the generalized Baues problem. This suggests natural extensions of objects related to the positive Grassmannian.

# 5 Posters

- Christopher Borger (Otto-von-Guericke-Universität Magdeburg):
  *Defectivity of families of point configurations*
- Paul Breiding (Max-Planck-Institut für Mathematik in den Naturwissenschaften):
  *Convex Geometry of Ricardian Trade Theory*
- Maxim Demenkov (Institute of Control Sciences):
  *Zonotopes in linear programming and projection of polytopes*
- Joseph Doolittle (University of Kansas):
  *Partition Extenders*
- Marek Filakovský (Masaryk University):
  *Computing simplicial representatives of homotopy group elements*
- Felix Günther (Technische Universität Berlin):
  *Discrete Riemann Surfaces*
- Harald Gropp (Universität Heidelberg):
  *Configurations in discrete geometry*
- Matjaz Kovse (IIT Bhubaneswar):
  *Topological Representation of the Transit Sets of $k$-Point Crossover Operators*
- Georg Loho (EPFL Lausanne):
  *Abstract tropical linear programming*
- Robert Löwe (Technische Universität Berlin):
  *Secondary fans and secondary polyhedra of punctured Riemann surfaces*
- Ivica Martinjak (Croatian Academy of Science and Art):
  *Inclusions and Classification Theorem of Finite Geometries*
- Jan Hendrik Rolfes (Universität zu Köln):
  *Geometric Coverings*
- Busra Sert (Mimar Sinan Fine Arts University):
  *On Minkowski Decomposition*
- Martin Skrodzki (Freie Universität Berlin):
  *Asymptotical and Combinatorial Results on the Neighborhood Grid Data Structure*