Variational methods for rate- and state-dependent friction

Elias Pipping[1] Oliver Sander[2] Ralf Kornhuber[1]

[1]Freie Universität Berlin: Institut für Mathematik

[2]RWTH Aachen University: Institut für Geometrie und Praktische Mathematik

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GeoΣim

HELMHOLTZ ASSOCIATION

CSS Center for Scientific Simulation

Freie Universität Berlin

Notation: Friction on a surface

$$\mu = \frac{|\sigma_t|}{|\sigma_n|}$$

where

μ: coefficient of friction

σ_t: tangential stress

σ_n: normal stress

Background: Experiments with rocks

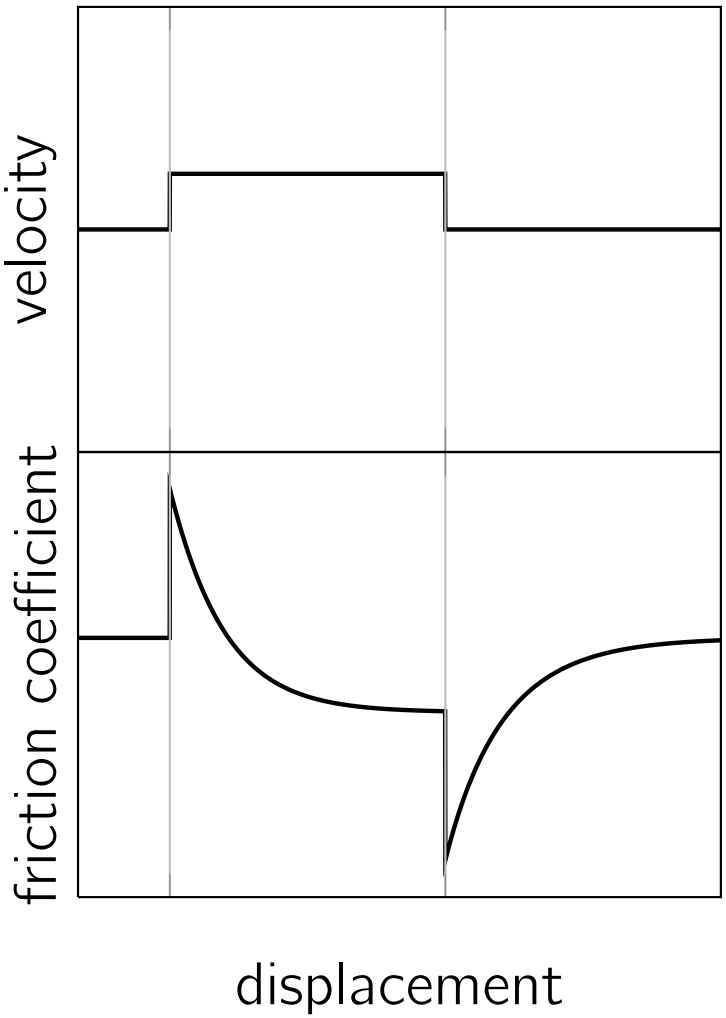


Figure: Idealised velocity-stepping test

Model: Rate- and state-dependent friction[1]

We assume

$$\mu = \mu(V, \theta)$$

where

$$V: \text{velocity (or “slip rate”)}$$
$$\theta: \text{material state}$$

and for the state

$$\dot{\theta} = \dot{\theta}(V, \theta)$$

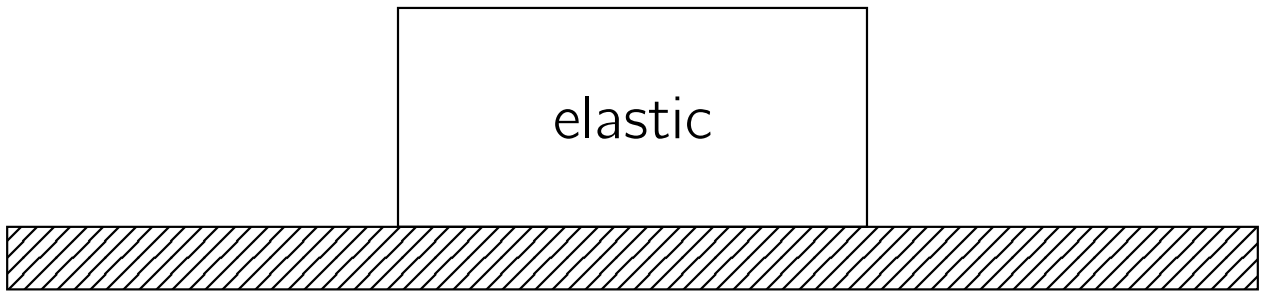
s.t. for fixed V

$$\theta \to \theta_{ss},$$
$$\mu \to \mu_{ss}.$$

[1] [Dieterich, 1979], [Ruina, 1983]

Our interest in this law

- frequently used model in geophysics
- elastic continua: little mathematical treatment
- analysis
  - What behaviour can we expect?
  - Are problems well-posed?
- numerics: fast and robust solvers



Analysis of the problem

Nice in some ways...

- $\mu(V, \theta)$ monotone in V for fixed θ.
  $\rightsquigarrow$ convex minimisation techniques.
- everything is smooth

...but not in others.

- coupled problem: θ and V are unknown.
- nonlinear PDE
- dissipation on the boundary only
  $\rightsquigarrow$ cannot use operator splitting

What's new? [1/2: Analysis]

- weak problem formulation
  $\rightsquigarrow$ two coupled time-dependent variational equations

[2] under certain assumptions on the model/parameters

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- time-discretisation
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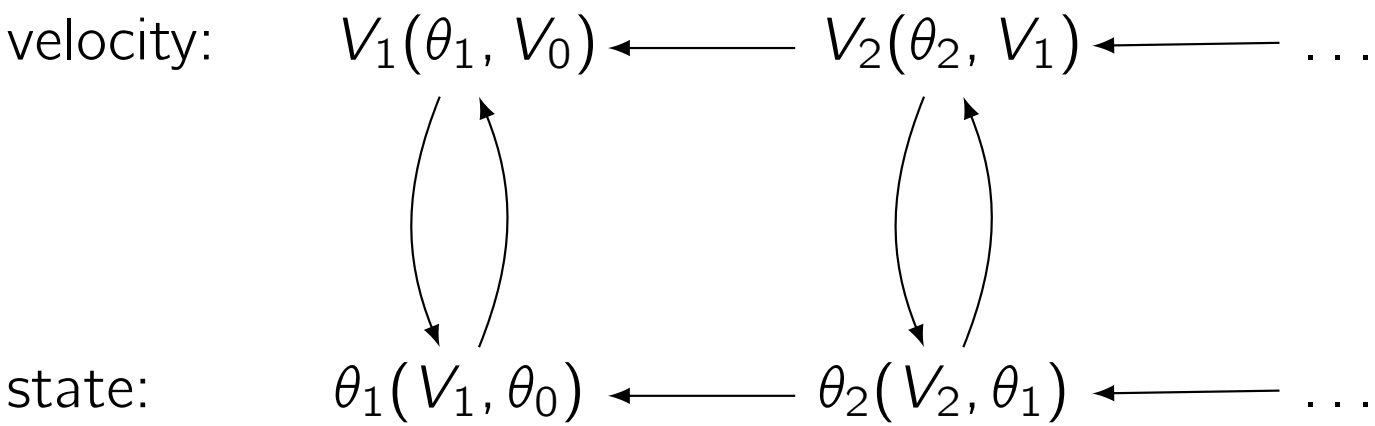

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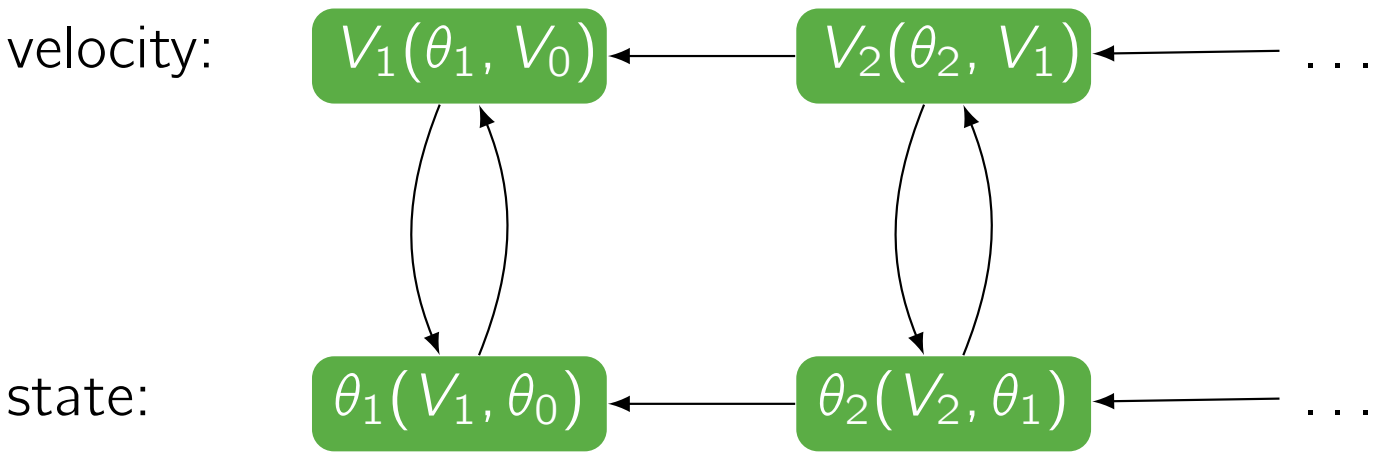
  - time-discrete subproblems: unique solution exists



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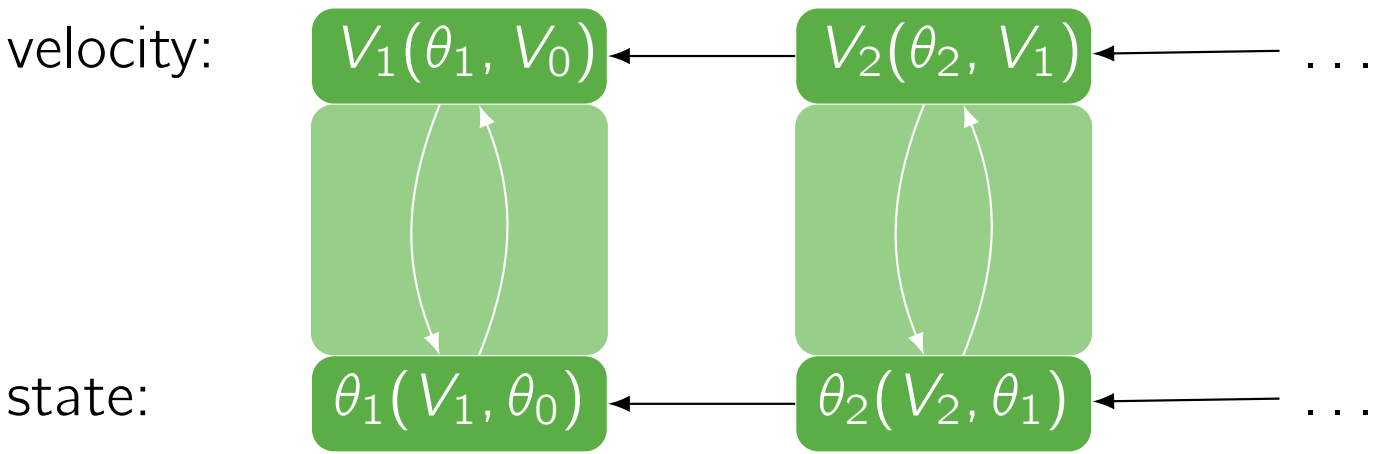
  - time-discrete subproblems: unique solution exists
  - time-discrete coupled problem: solution exists[2]
    by Schauder's fixed point theorem (no uniqueness)



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What's new? [2/2: Numerics]

- algorithm based on Truncated Nonsmooth Newton Multigrid[3] & fixed-point iteration.
- implementation based on the DUNE-framework[4]
  - relaxed fixed point iteration converges
  - number of fixed point iterations stable

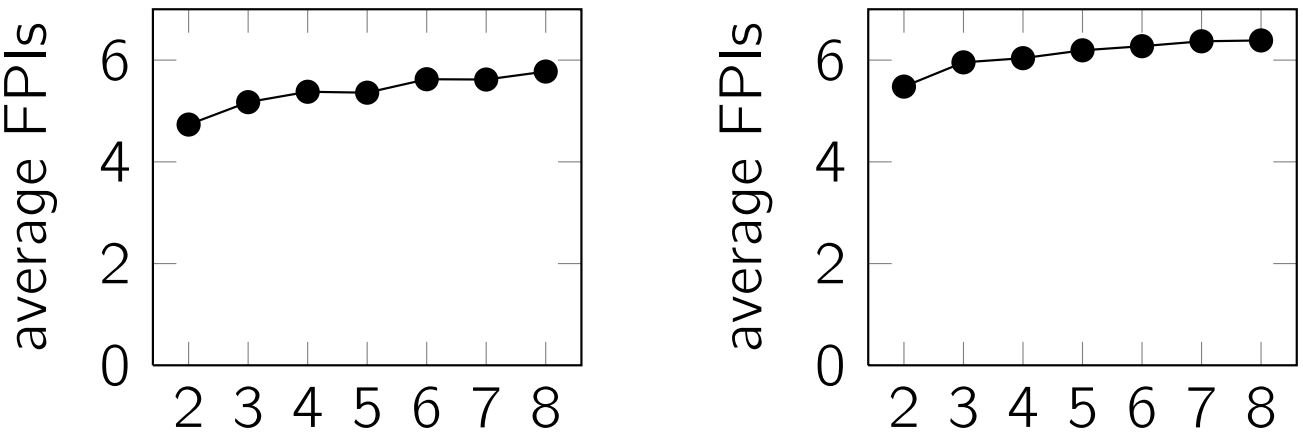

Figure: average number of fixed point iterations per time step

[3] [Gräser et al., 2009]

[4] Distributed Unified Numerics Environment, `http://dune-project.org`

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